Potential-free Nuclear Physics, Connecting Lattice QCD to Nuclear Physics

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Introduction

- The original Harmonic Oscillator Based Effective Theory (HOBET) work by Haxton and Luu reduced $H = T + V_{NN}$, with V_{NN} a realistic potential, to H^{eff} in a small basis defined by projection operator P while correctly including all scattering by H through an excluded space Q. Scattering by T is analytically included to all orders, leaving the effective theory (ET) expansion solely focused on on the short range V_{NN}.
- We embrace energy dependence in the effective interaction as it is key to accounting for scattering through the excluded Q space. T is tridiagonal in a harmonic oscillator basis and strongly couples the P and Q subspaces. Much of the energy dependence of H^{eff} comes from this coupling.
- Results fundamentally do not depend on the choice or size of the included space defined by P, or on the choice of the harmonic oscillator length scale $b = \sqrt{\hbar/(M\omega)}$ with M the nucleon mass. This is because all scattering by H through the excluded space Q is incorporated into the interaction. This distinguishes HOBET from other effective theory approaches which take harmonic oscillator matrix elements of a momentum basis effective theory of the interaction. The resulting momentum and harmonic oscillator cutoffs overlap, resulting in uncontrolled error.
- Here we abandon the potential V_{NN} and instead determine the low energy constants (LECs) of the effective theory expansion from energy eigenvalues and boundary conditions. In the infinite volume continuum case every energy is an eigenvalue of H with a boundary condition at infinity corresponding to the phase shift. One connection to LQCD is that Lüscher's method can be used to relate finite volume eigenvalues to infinite volume scattering phase shifts.

Fitting the HOBET LECs to Phase Shifts

The first step is the reorganization of the Bloch-Horowitz equation to separate the long range kinetic energy contribution from the assumed short range potential.

Infrared Lee-Suzuki Operator

$$H_{eff} = P\frac{E}{E-TQ}\left[T - \underbrace{T\frac{Q}{E}T}_{} + V + \underbrace{V\frac{1}{E-QH}V}_{}\right]\frac{E}{E-QT}P$$

Sums Scattering by T Through Q to All Orders

Capped by V, and Therefore Short Range. Replace with a Contact Operator Expansion δV

Figure: Pieces of the Haxton-Luu form of the Bloch-Horowitz equation.

The Infrared Lee-Suzuki operator plays a key role in restoring the long range behavior of the wave function. It's action is limited to end-shell states which boarder Q. When acting on the end-shell states $G_{QT} = E/(E - QT)$ recovers the long range behavior of eigenstates of H through matching the Green's function to the phase shifted wave function at infinity.

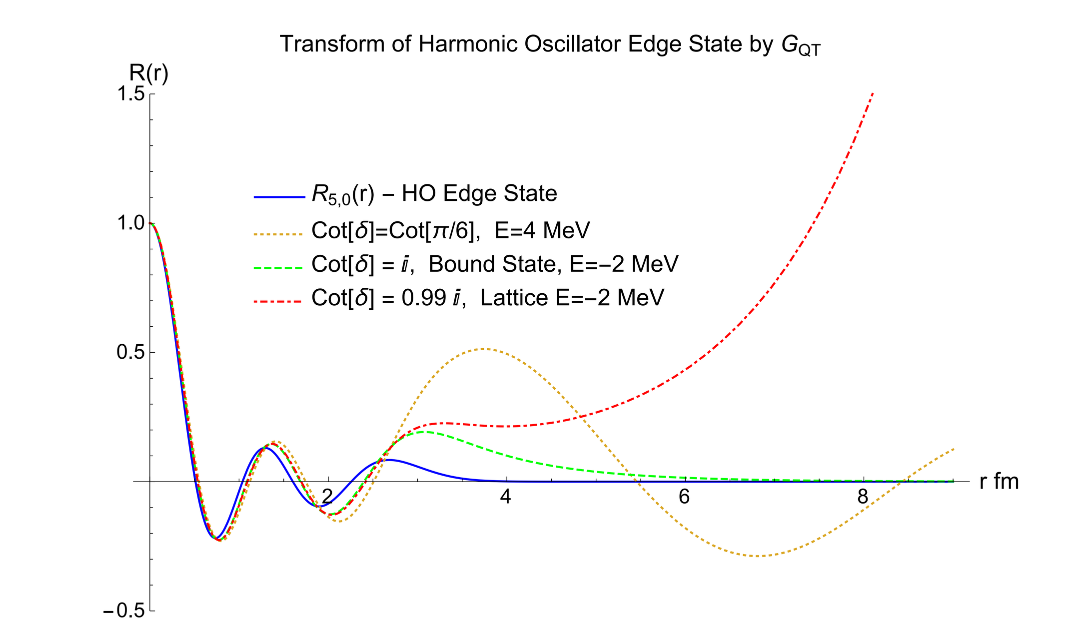


Figure: Transform of the end-shell state under three boundary conditions corresponding to a scattering state, a bound state and the divergent infinite volume state produced by Lüscher's formula for an LQCD state. All wave functions are scaled to match at the origin.

To constrain the Green's function we note that $\frac{E}{E-QT}P = \frac{E}{E-T}\left\{P\frac{E}{E-T}P\right\}^{-1}P$, which makes it clear that constraining $G_0 = E/(E-T)$ to match the boundary condition will result in G_{QT} also matching it.

V and δV play synergistic roles. In the original sense of the Bloch-Horowitz equation V would be the full potential and δV would capture the effect of renormalization, or integrating out the excluded Q space. Since we will be fitting the expansion of δV to match the energy eigenvalues it can also fit short range parts of V at the same time. The function we use for V should then match the longer range behavior of the full potential because that is harder to fit at finite order with the δV expansion. We use a one pion exchange potential for V with the pion-nucleon coupling constant determined from the fit.

The δV expansion is written in terms of SHO lowering operators with $\hat{b}$ lowering the nodal quantum number and $\left[\hat{c}^2\right]_2$ a rank 2 tensor operator lowering L by 2. $d_{n',n}$ are matrix elements of the delta function. We show below an expansion for the S channel and tensor interaction between S and D.

$$\begin{aligned}
V_\delta^{\mathrm{S}} = \sum_{n'n} d_{n'n} \Big[& a_{LO}^{\mathrm{S}} |n'0\rangle\langle n0| + a_{NLO}^{\mathrm{S}} \left\{\hat{b}^\dagger |n'0\rangle\langle n0| + |n'0\rangle\langle n0|\hat{b}\right\} + a_{NNLO}^{\mathrm{S},22}\, \hat{b}|n'0\rangle\langle n0|\hat{b} \\
& + a_{NNLO}^{\mathrm{S},40} \left\{\hat{b}^{\dagger 2}|n'0\rangle\langle n0| + |n'0\rangle\langle n0|\hat{b}^2\right\} + a_{N^3LO}^{\mathrm{S},42} \left\{\hat{b}^{\dagger 2}|n'0\rangle\langle n0|\hat{b} + \hat{b}^\dagger|n'0\rangle\langle n0|\hat{b}^2\right\} \\
& + a_{N^3LO}^{\mathrm{S},60} \left\{\hat{b}^{\dagger 3}|n'0\rangle\langle n0| + |n'0\rangle\langle n0|\hat{b}^3\right\} \Big] \\
V_\delta^{\mathrm{SD}} = \sum_{n'n} d_{n'n} \Big[& a_{NLO}^{\mathrm{SD}} \left\{ \left[\hat{c}^{\dagger 2}\right]_2 |n'0\rangle\langle n0| + |n'0\rangle\langle n0| \left[\hat{c}^2\right]_2 \right\} + o(\mathrm{NNLO}) + o(\mathrm{N^3LO}) \Big] \odot [\boldsymbol{\sigma}_1 \otimes \boldsymbol{\sigma}_2]_2
\end{aligned}$$

Application to Neutron Proton Scattering : The $^3P_0$ Channel

For testing purposes we use the Argonne v_{18} potential as a reference to generate phase shifts and reference wave functions for comparison. For V we use an OPEP with coupling constant fit in the $^1F_3$ channel. The low energy constants (LECs) of the effective theory expansion of δV are fit to minimize the sum of squared relative error of the effective Hamiltonian eigenvalues across the shown sample energies (1 to 80MeV). We use an included space with an energy cutoff $\Lambda = 8$ (4 states) and length scale for the harmonic oscillator basis of $b = 1.7 fm$.

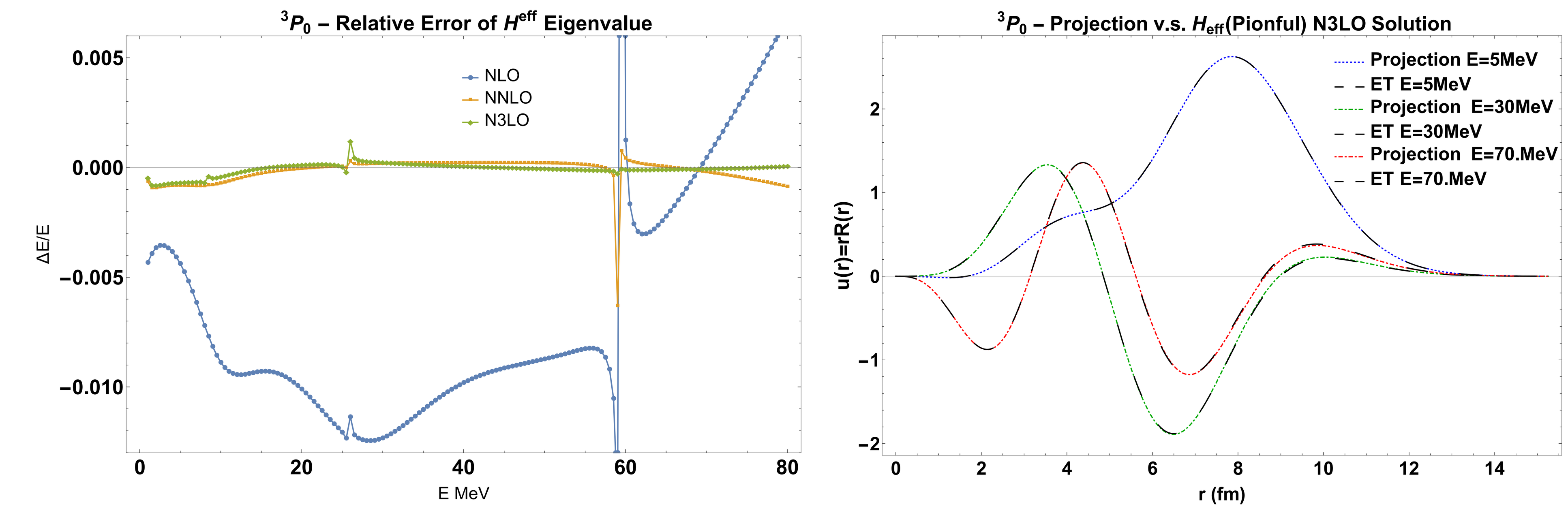


Figure: Lepage Plot showing Convergence and Projected v.s. ET Wave Functions at 3 energies in a P space with 4 states.

On the left we see the convergence of the relative energy self consistency error with the order of the ET theory. On the right the restriction of the full numerical solution to P and the ET wave functions are nearly identical at N3LO.

The Deuteron Bound State - $^3S_1/^3D_1$ Coupled Channel

The deuteron bound state is found with P defined by $\Lambda = 8$, containing 5 $^3S_1$ and 4 $^3D_1$ states. The ET LECs are fit to produce the required self consistent energies across the 1 to $40\mathrm{MeV}$ range.

Order	$E_{bind}^{pionless}$	$C^2(LECs)$	$E_{bind}^{pionful}$	$C^2(LECs)$
bare	3.09525	-	-0.76775	-
LO	-1.27715	2.2E-2	-2.01110	1.9E-3
NLO	-1.95424	1.6E-2	-2.19833	2.2E-6
NNLO	-2.17307	6.7E-3	-2.21705	4.0E-8
N^3LO	-2.23175	1.3E-3	-2.22464	8.4E-9

Table: Deuteron binding energy convergence. In the pionless case we set $V = 0$ and in the pionful case $V = V_\pi$.

To obtain similar convergence to the binding energy of $-2.2245\mathrm{MeV}$ with matrix elements of Av_{18} one requires a large basis with $\Lambda > 150$.

New: HOBET in a Box - Directly Connecting to LQCD States

HOBET in a Cartesian basis is a good match to LQCD NN eigenstates. Phase shift based boundary conditions are replaced with periodic boundary conditions and a Cartesian ET expansion can be rewritten in terms of the spherical LECs!

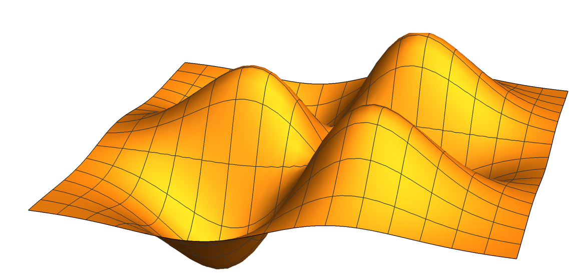
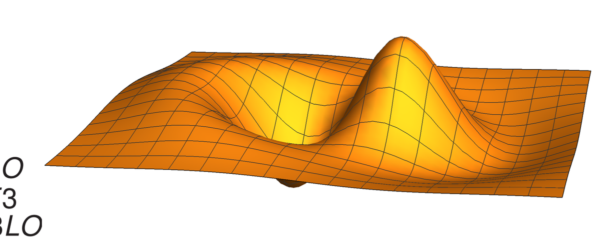


Figure: CHO state $\vec{n} = 1, 2, 0$ in x,y plane

Cartesian	Spherical
$c000d000 =$	a_{LO}^{1S0}
$c200d200 =$	$a_{NNLO22}^{1S0} + (2/3) * a_{NNLO}^{1D2}$
$c300d120 =$	$a_{N3LO22}^{1P1} - (6/10) * a_{N3LO}^{1F3}$
$c111d111 =$	$6 * a_{N3LO}^{1F3}$

Cartesian LECs expanded in Spherical LECs



SHO n=2,P,m=0 in y,z plane.

The non-physical angular momentum state mixing encountered in LQCD work is sequestered in G_{QT}, which implements the boundary conditions. Once the LECs are fit to energy eigenvalues, simply swapping boundary conditions produces the infinite volume angular momentum decomposition of the interaction! This bypasses Lüscher's method and we believe that it will much easier to generalize to 3-body interactions.

Conclusions

- We have demonstrated construction of the nuclear effective interaction without the construction of an intermediate potential from energy eigenstates and boundary conditions.
- The resulting effective theory faithfully implements integrating out the excluded Q space, yielding accurate convergence in small P spaces, distinguishing this work from other approaches.
- A Cartesian version of HOBET with periodic boundary conditions, appropriate for LQCD NN eigenstates, can be linked to the infinite volume angular momentum channel form, giving a path to determining properties of nuclei directly from QCD.

Acknowledgements

- Support for this work was provided by the DOE under contract DE-AC02-05CH11231.
- Support for this work was provided through Scientific Discovery through Advanced Computing (SciDAC) program funded by U.S. Department of Energy, Office of Science, Advanced Scientific Computing Research and Nuclear Physics.